

# Self-Regulated Star Formation and the Black Hole – Galaxy Bulge Relation

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## ABSTRACT

We show that star formation in galaxy bulges is self-regulating through momentum feedback, limiting the stellar bulge mass to  $M_b \propto \sigma^4$ . Together with a black hole mass  $M_{\text{BH}} \propto \sigma^4$  set by AGN momentum feedback, this produces a linear  $M_{\text{BH}} - M_b$  relation. At low redshift this gives  $M_{\text{BH}}/M_b \sim 10^{-3}$ , close to the observed ratio. We show that AGN feedback can remove any remaining gas from the bulge and terminate star formation once the central black hole reaches the  $M_{\text{BH}} - \sigma$  value, contrary to earlier claims. We find a mild upward deviation from the  $\sigma^4$  law at higher redshift and at higher  $\sigma$ .

**Key words:** accretion: accretion discs – galaxies: formation – galaxies: active – black hole physics

## 1 INTRODUCTION

It is now widely accepted that the centre of every medium-to-large galaxy contains a supermassive black hole (SMBH). The mass  $M$  of this SMBH is observed to correlate strongly with properties of the host galaxy in two ways. First is the correlation with the stellar velocity dispersion  $\sigma$  of the galaxy bulge,

$$M_{\text{BH}} \simeq 1.5 \times 10^8 \sigma_{200}^4 M_{\odot} \quad (1)$$

(cf. Ferrarese & Merritt 2000; Gebhardt et al. 2000), where  $\sigma_{200} = \sigma/200 \text{ km s}^{-1}$ . Second is the correlation with bulge stellar mass  $M_b$

$$M_{\text{BH}} \simeq 1.6 \times 10^{-3} M_b \quad (2)$$

(cf. Häring & Rix 2004). The first of these relations (1) is considerably tighter than the second (2), but both are highly significant statistically.

The existence of these relations strongly suggests a connection between the growth of the SMBH and of its host galaxy, despite the huge disparity in their masses. The underlying reason is evidently that the accretion energy  $E_h = \epsilon_{\text{acc}} M c^2$  released in building the SMBH (where  $\epsilon_{\text{acc}} \sim 0.1$  is the energy efficiency of accretion) greatly exceeds the binding energy of the bulge  $E_b \sim M_b \sigma^2$  by a factor

$$\frac{E_h}{E_b} \sim \frac{360}{\sigma_{200}^2}. \quad (3)$$

Thus absorption of even a small fraction of the accretion energy released by the SMBH must significantly affect the bulge (King 2003). This suggests that the process connecting SMBH growth and galaxy growth is some kind of feedback from the SMBH. Because galaxies are generally optically thin this feedback must be mechanical, and so presumably involves matter expelled during the accretion process. This in turn strongly suggests that feedback is a consequence of super-Eddington accretion. This is plausible because the Soltan (1982) relation linking the growth of SMBHs to luminous accretion, and the low observed fraction of active galaxies among all galaxies (e.g. Heckman et al. 2004), together require that most SMBH growth occurs in such phases.

The remaining obstacle to explaining the  $M_{\text{BH}} - \sigma$  relation (1) is the very inefficient mechanical coupling of accretion energy implied by (3). This explains why the proposal by Silk & Rees (1998), that a significant fraction of a SMBH's Eddington energy rate

$$L_{\text{Edd}} = \frac{4\pi G M_{\text{BH}} c}{\kappa} \quad (4)$$

(where  $\kappa$  is the Thomson opacity) is deposited as feedback into the surrounding galaxy, implies SMBH masses of order only  $\sim 10^4 - 10^5 M_{\odot}$  rather than  $\sim 10^8 M_{\odot}$  for a typical galaxy (cf. King 2010). Instead, two further features of the problem imply a theoretical relation very close to the observed one (1),

$$M_{\text{BH}} = M_{\sigma} = \frac{f_g \kappa}{\pi G^2} \sigma^4, \quad (5)$$

where  $f_g \simeq 0.16$  is the baryon fraction relative to dark matter (cf. King 2003, 2005). These new features are (a)

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the observation that SMBH accretion is never highly super-Eddington, so that each emitted photon scatters on average only once, implying that the wind from the SMBH region carries momentum at the Eddington rate  $L_{\text{Edd}}/c$ ; and (b) in impacting the interstellar medium of the host bulge, the wind shocks and loses almost all its energy through Compton cooling by the AGN radiation field. As King (2010) notes, feature (a) implies that we can treat the problem in the single-scattering limit, while feature (b) implies that the wind impact creates a *momentum-driven* outflow in the host ISM.<sup>1</sup>

It follows that, for SMBH masses  $M_{\text{BH}} < M_\sigma$ , an outflow from the vicinity of the SMBH vicinity will remain trapped very close to it. Once  $M_{\text{BH}}$  reaches the critical value  $M_\sigma$ , the momentum rate  $L_{\text{Edd}}/c$  of the wind precisely balances the total weight  $4f_g\sigma^4/G$  of the overlying gas (which, assuming isothermal mass distributions for both the dark matter and baryons, is independent of radial extent; cf. Nayakshin, Wilkinson, & King 2009). The equality between thrust and weight then gives the theoretical relation (5), as the outflow propagates through the bulge on its dynamical time and presumably halts further accretion on to the SMBH. Precisely how this last step occurs is still the most unclear part of any theory of the  $M_{\text{BH}} - \sigma$  relation (see, for example, Nayakshin & Power 2010).

Self-regulated SMBH growth may explain the  $M_{\text{BH}} - \sigma$  relation, but what of the  $M_{\text{BH}} - M_b$  relation (2)? As noted in King (2003), Compton cooling becomes inefficient beyond a certain radius  $R_c$  within the bulge. At a SMBH mass  $M_{\text{BH}} = M_\sigma$  the shock rapidly reaches this radius and then accelerates to a higher speed because of the thermal expansion of the shocked wind, which now no longer cools. Thus the outflow changes its character from momentum-driven to energy-driven at this point.

This high-speed outflow removes any ambient gas from the bulge, quenching star formation in the process, and so it is tempting to identify the original baryonic mass within this radius as the bulge mass  $M_b$ , as was argued by King (2003) and King (2005)). However, such an identification is not entirely satisfactory because the relation of the original gas mass to the expected stellar mass of the bulge in this model remains unclear – it may be all or nothing. The goal of this short paper is to address this deficiency of the model.

We first recall the results of McLaughlin, King, & Nayakshin (2006), who pointed out that star formation in the nuclear star cluster (NC) of a galaxy may provide enough momentum thrust to establish its own  $M_{\text{NC}} - \sigma$  relation, offset from the SMBH  $M_{\text{BH}} - \sigma$  relation. They emphasised that the momentum feedback produced by young NCs is actually quite similar to that produced by a SMBH accreting at its Eddington

rate but smaller by a factor of  $\sim 20$  for a Chabrier (2003) IMF. This similarity arises because massive young stars are observed to radiate near their Eddington limit and to produce outflows at rates almost matching that of a black hole of the same mass in the model of King (2003, 2005).

This paper extends the idea of McLaughlin, King, & Nayakshin (2006) by investigating momentum feedback from a galaxy’s stellar bulge, rather than its central NC. In particular, we explore the role stellar feedback could play in regulating bulge mass  $M_b$  and in shaping the  $M_{\text{BH}} - M_b$  relation. We note that Murray et al. (2005) followed a similar approach in their explanation of the origin of the Faber & Jackson (1976) relation ( $L \propto \sigma^4$ ) for elliptical galaxies. There is an important distinction, however, between feedback from a galaxy’s bulge and feedback from its NC. McLaughlin, King, & Nayakshin (2006) considered *instantaneous* feedback from young massive stars in a NC while they are still on their main sequence. In contrast, the shortest timescale in a galaxy bulge is the dynamical timescale, which is much longer than the lifetimes of massive stars (cf. Nayakshin, Wilkinson, & King 2009), and so we must treat feedback from the bulge in a time-integrated or *extended* form. As for NCs, the feedback produced in this case is proportional to the total mass of the bulge – but with a smaller efficiency because the stellar population in the bulge is relatively mature.

We explore these ideas in more detail in the following sections. We begin by discussing star formation and the role it plays in regulating the masses of the galaxy bulges (§ 2); then we examine how gas is swept out of the bulge by black hole feedback (§ 3); and finally, we assess the implications of these ideas for the  $M_{\text{BH}} - M_b$  relation (§ 4).

## 2 STAR FORMATION IN GALAXY BULGES

We begin our analysis by considering how star formation in a galaxy bulge impacts on the mass of virialised gas  $M_{\text{g,vir}}$  in a dark matter halo. A fraction of the virialised gas mass will be converted into bulge stars ( $M_b$ ), but once the stellar mass in the bulge exceeds a critical value ( $M_{\text{max}}$ ), momentum-driven feedback from the stars (in the form of winds and supernovae) will expel this gas and help to regulate the bulge mass by suppressing further star formation. Our goal is to establish what mass of stars must form to suppress further star formation. As we noted in the introduction, a similar problem has been explored by Murray et al. (2005) as an explanation for the origin of the Faber & Jackson (1976) relation for elliptical galaxies,  $L \propto \sigma^4$ .

Massive stars dominate the mass and momentum outflow from star clusters through supernovae and stellar winds (cf. Leitherer, Robert & Drissen 1992; Lamers & Cassinelli 1999). For a Salpeter (1955) initial mass function (IMF) and solar metallicity, the contributions peak at stellar masses of  $12M_\odot$  for supernovae and about  $50M_\odot$  for momentum outflow via winds. Integrated over time, these two feedback mechanisms contribute roughly equally to the momentum outflow from massive star clusters. Consequently, the characteristic main sequence age for stars dominating momentum feedback is  $t_{\text{ms}} \simeq 10^7$  years. This is of order the dynamical time in inner parts of the dark matter halo and shorter than the Salpeter timescale ( $t_s \simeq 4 \times 10^7$  years) on which

<sup>1</sup> Similar arguments are presented in Murray et al. (2005), who obtain a result identical to (5). However, they assume that it is dust rather than electron scattering that drives the outflow, which implies multiple rather than single photon scatterings. The resulting momentum flux scales as  $\tau L/c$ , where  $\tau$  is the optical depth, and so a SMBH that is formally sub-Eddington in the electron scattering sense can radiate effectively at the SMBH’s Eddington limit.

the SMBH grows. This means that the rate of stellar momentum feedback is fixed by the rate at which new massive stars form and replace ones that are dying or have died (see Fig. 12 in Leitherer, Robert & Drissen 1992). In this regime, star formation injects momentum to the ambient medium at the global rate

$$\dot{p}_* \simeq \epsilon_* c \dot{M}_* \quad (6)$$

where  $\epsilon_* \simeq 10^{-3}$  (cf. Leitherer, Robert & Drissen 1992; see also §2.2 of Murray et al. 2005).

Hence the total momentum produced by a stellar mass  $M_*$  is simply

$$p_* \simeq \epsilon_* c M_*. \quad (7)$$

Feedback may inhibit star formation when gas that is not locked into stars acquires a typical velocity of order the velocity dispersion  $\sigma$ . Let  $M_0$  be the initial bulge gas mass prior to the onset of star formation; we assume that  $M_0$  is of order the virial value  $M_{g,\text{vir}}$ . Feedback inhibits further star formation in the bulge when the total momentum injected into bulge gas reaches

$$p_{\text{inh}} \simeq \eta M_0 \sigma, \quad (8)$$

where  $\eta \sim 1$ .  $\eta$  captures the uncertainty in the amount of momentum feedback that the system can absorb before star formation is suppressed. For example, if the gas needs to be completely expelled from the halo by star formation feedback rather than by SMBH feedback, then we might expect  $\eta$  to be greater than unity because the average escape velocity from the galaxy will be typically just above  $\sigma$ . On the other hand, if SMBH expels the remaining gas then  $\eta$  could be less than unity: star formation feedback may simply need to slow down collapse of gas into stars until the SMBH grows up to the  $M_{\text{BH}} - \sigma$  value (e.g. Murray et al. 2005; Nayakshin, Wilkinson, & King 2009). Observational evidence can provide important guidance as to what the value of  $\eta$  might be, as we discuss below.

The derivation of the  $M_{\text{BH}} - \sigma$  relation in King (2005) shows that mechanical feedback from accretion onto the SMBH is confined to a small region near it initially. This suggests that stellar feedback is the most plausible mechanism by which star formation can be inhibited up until this point (see also § 5 of Murray et al. 2005). Therefore, this means that the total stellar bulge mass  $M_b$  cannot exceed a value  $M_{\text{max}}$  such that  $p_* = p_{\text{inh}}$ ; this means that

$$\epsilon_* c M_{\text{max}} \simeq \eta M_0 \sigma \quad (9)$$

from (7), which gives

$$M_b \lesssim M_{\text{max}} \simeq \eta M_0 \frac{\sigma}{\epsilon_* c}, \quad (10)$$

or

$$M_b \lesssim M_{\text{max}} \simeq 0.6 \eta M_0 \sigma_{200}; \quad (11)$$

where  $\sigma_{200} = \sigma / 200 \text{ km s}^{-1}$ . For the moment we assume that  $\eta = 1$  but we keep in mind our discussion from above.

### 3 SWEEPING OUT THE BULGE

Inspection of relation (11) suggests that a significant gas mass  $M_{\text{gas}}$  might be present in a galaxy bulge as the stellar mass asymptotes towards its limiting value  $M_b$ ; we would

expect  $M_{\text{gas}}$  to be of order the baryon virial mass  $M_{g,\text{vir}}$ . The observed  $M_{\text{BH}} - M_b$  relation then implies that feedback from the central SMBH must be able to sweep away this gas as  $M_{\text{BH}}$  reaches the value  $M_\sigma$  given by the  $M_{\text{BH}} - \sigma$  relation (5). As the escape speed from an isothermal bulge is  $\simeq 2\sigma$ , the SMBH must supply an energy

$$E_{\text{esc}} = \frac{1}{2} M_{\text{gas}} (2\sigma)^2 \quad (12)$$

to this gas.

To check this, we assume that the wind driven by super-Eddington accretion on to the SMBH provides thrust given by the single-scattering limit, i.e.

$$\dot{M}_{\text{out}} v \simeq \frac{L_{\text{Edd}}}{c}, \quad (13)$$

where  $\dot{M}_{\text{out}}$  is the wind mass outflow rate and  $v$  is its speed. This limit implies that the wind momentum is similar to the original photon momentum, i.e. the scattering optical depth is  $\sim 1$  and each photon scatters on average about once before escaping (cf. Lamers & Cassinelli 1999). This is likely to hold for SMBH accretion because even the dynamical accretion rate is not much larger than the Eddington rate  $\dot{M}_{\text{Edd}}$  for the hole near its  $M - \sigma$  mass. That this is so is shown explicitly in King (2010), which also shows that

$$v \simeq \epsilon_{\text{acc}} c \quad (14)$$

in these circumstances, where  $\epsilon_{\text{acc}} \simeq 0.1$  is the accretion efficiency. Accordingly the total energy of the outflow is

$$E_{\text{out}} = \frac{1}{2} \Delta M_{\text{BH}} v^2 = \frac{1}{2} \Delta M_{\text{BH}} (\epsilon_{\text{acc}} c)^2, \quad (15)$$

where  $\Delta M_{\text{BH}}$  is the increase in SMBH mass during the accretion episode considered. The fraction of  $E_{\text{out}}$  supplied to the bulge gas now depends on the nature of the shock as the wind impacts on it. If the shocked gas cools rapidly (faster than the flow time), the outflow is *momentum-driven* (e.g. Dyson & Williams 1997) and only a small fraction of  $E_{\text{out}}$  is used to drive the outflow. However, this turns out to be slightly too small to expel it (as we show in the appendix). In contrast, if the shocked gas does not readily cool (an *energy-driven* outflow), the thermal expansion of the shocked gas means that almost all of  $E_{\text{out}}$  is transferred to the gas to expel it.

King (2003, 2005) has shown that the outflow is momentum-driven when very close to the SMBH, but energy-driven outside a radius  $\sim 1$  kpc for a typical galaxy bulge<sup>2</sup>. This is because the efficiency with which the quasar radiation field Compton cools the shock preceeding the outflow declines with increasing radius (cf. Ciotti & Ostriker 1997). If we require that the thrust of the momentum-driven outflow must exceed the weight of the overlying gas, as must be the case if the outflow is to escape from the immediate vicinity of the black hole, then it is straightforward to show

<sup>2</sup> The precise value of this radius scales as  $(v/c)^2$ . This can be seen by comparing the flow time of the outflow to the Compton cooling time in the shock preceeding the outflow, given by equations (9) and (8) respectively of King (2003), with equation (14) for the flow velocity corrected by a factor of 2 to be consistent with the value of  $M_{\text{BH}}$  derived in King (2005). In our analysis we assume a wind velocity of  $\sim 0.1c$ , which is consistent with observational data (cf. Figure 8 of Tombesi et al. 2010).

that a  $M_{\text{BH}} - \sigma$  of the form given by equation (5) must follow.

However the total gas mass that the outflow can ultimately remove is given by considering the total energy  $E_{\text{out}}$ , as the outflow rapidly becomes energy-driven. By equating  $E_{\text{out}}$  and  $E_{\text{esc}}$  we find that

$$\frac{\Delta M_{\text{BH}}}{M_{\text{gas}}} = \left( \frac{2\sigma}{\epsilon_{\text{acc}} C} \right)^2 = 1.8 \times 10^{-4} \sigma_{200}^2 \quad (16)$$

i.e. only a small increase in SMBH mass is needed to sweep the bulge clear of any remaining gas. We quantify this further below.

#### 4 THE BLACK HOLE – BULGE MASS RELATION

We have argued that momentum-driven outflows from star formation tend to produce a bulge stellar mass

$$M_b \lesssim 0.6\eta M_{\text{g,vir}} \sigma_{200}. \quad (17)$$

Here we assume that  $M_0$  in equation (11) is of order the virialised gas mass within the dark matter halo, i.e.  $M_{\text{g,vir}} = f_g M_v$ , where  $M_v$  is the total virial mass, and  $\eta$  captures the uncertainty in the amount of momentum feedback the system can absorb before star formation is suppressed. For the sake of simplicity, we also assume that both the gas and dark matter follow isothermal mass distributions, and that the velocity dispersion  $\sigma$  of the bulge and its underlying dark matter halo are the same. Neither of these assumptions are likely hold in detail, but their effect is quantitative rather than qualitative. Assuming isothermality, how does the virial gas mass  $M_{\text{g,vir}}$  vary with velocity dispersion  $\sigma$ ?

We make the standard assumption that matter is virialised within a radius such that the mean density is 200 times the critical value; for the particular case of an isothermal sphere, this gives

$$R_v = \frac{\sigma}{5\sqrt{2}H} \quad (18)$$

where  $H(z) = 100 h(z) \text{ kms}^{-1} \text{ Mpc}^{-1}$  is the Hubble parameter at the given redshift  $z$ , with  $h(z)$  the dimensionless Hubble parameter. Because  $M_v = 2\sigma^2 R_v / G$  we find that

$$M_b \lesssim 5.1 \times 10^{11} \eta \frac{f_g}{0.16} \frac{\sigma_{200}^4}{h(z)} M_{\odot}; \quad (19)$$

when combined with equation (5), we find that

$$\frac{M_{\sigma}}{M_b} \gtrsim \frac{7.3 \times 10^{-4} h(z)}{\eta}. \quad (20)$$

in reasonable agreement with, for example, Häring & Rix (2004).

We expect deviations from this relation and the simple  $M_{\text{BH}} \sim \sigma^4$  law, as  $M_{\text{BH}}$  may grow above  $M_{\sigma}$  by an amount  $\lesssim \Delta M_{\text{BH}}$  in expelling any remaining gas. Combining (20) with (16) – where we assume that  $M_{\text{gas}}$  is equal to  $M_{\text{g,vir}}$  – and (17) gives the predicted maximum deviation from the theoretical relation

$$\frac{\Delta M_{\text{BH}}}{M_{\sigma}} \lesssim 0.41 \frac{\sigma_{200}}{h(z)}. \quad (21)$$

This implies that  $M_{\text{BH}}$  should tend to increase above  $M_{\sigma}$  with redshift and with  $\sigma$ . With  $M_{\text{BH}} = M_{\sigma} + \Delta M_{\text{BH}}$  the black hole – bulge mass relation finally becomes

$$\frac{M_{\text{BH}}}{M_b} \sim \frac{7.3 \times 10^{-4} h(z)}{\eta} \left[ 1 + \frac{0.41 \sigma_{200}}{h(z)} \right] \quad (22)$$

Assuming  $h = 0.7$  at  $z = 0$ , this implies that

$$\frac{M_{\text{BH}}}{M_b} \sim 1.15 \times 10^{-3} \eta^{-1} \quad (23)$$

for a galaxy with  $\sigma_{200} = 1$ , close to the observed relation (2). This relation should curve slightly upwards for larger  $\sigma$ .

#### 5 DISCUSSION

We have assumed that star formation in galaxy bulges is self-regulating through momentum feedback acting on gas in the galaxy’s dark matter halo. This puts a limit on the total mass of the stars that can form within the halo (equation 17). Crucially, this maximum bulge mass scales as  $\sigma^4$  with the galaxy velocity dispersion. Given that the SMBH mass also scales as  $\sigma^4$  (cf. King 2003, 2005; Murray et al. 2005), this results in a linear  $M_{\text{BH}} - M_b$  relation, close to the observed one (Häring & Rix 2004). This relation arises because both the SMBH and the bulge are limited by essentially the same quantity – the maximum momentum thrust that the system can take before the gas is blown away.

The best match to the observed relation (Häring & Rix 2004) is obtained for the dimensionless parameter  $\eta \sim 0.7$ . In the context of our order of magnitude derivation in §2, a value of  $\eta$  less than one implies that the bulge is not entirely self-sufficient in limiting its own growth, and that a SMBH or a NC are required to terminate bulge growth completely.

Finally, we argued in §4 that if the SMBH growth timescale (few  $\times$  Salpeter) is shorter than the star formation (dynamical) timescale, the growth of the central black hole is able to shut off star formation in the bulge early. In this case the extra SMBH growth introduces a mild upward trend in the  $M_{\text{BH}} \propto \sigma^4$  relation.

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## APPENDIX

If one assumes that the outflow is momentum-driven at all radii (i.e. efficient shock cooling everywhere), the outflow would exert a thrust given precisely by the weight  $4f_g\sigma^4/G$  of the overlying gas of mass  $M_{\text{gas}}$ . Then the total work done by this thrust on the gas, assumed to extend to some radius  $R$ , is

$$E'_{\text{out}} = \frac{4f_g\sigma^4 R}{G} = 2\sigma^2 M_{\text{gas}} = \frac{1}{2}E_{\text{esc}} \quad (24)$$

because the gas is in equilibrium with the isothermal dark matter halo. Therefore a purely momentum-driven outflow would fail by a factor 2 to remove the overlying gas, which is essentially the result claimed by Silk & Nusser (2010).

However, cooling is ineffective once the shock radius is  $\gtrsim 1$  kpc for a typical galaxy and so the outflow becomes energy-driven. This is because the shock is Compton cooled while it is close to the SMBH and the radius at which the Compton cooling time begins to exceed the shock flow time varies as  $0.75\sigma_{200}^3$  kpc, where we've assumed a wind velocity of  $0.1c$ . If the shock is to cool radiatively, it must do so by means of thermal bremsstrahlung because of the magnitude of the shock temperature ( $T \simeq 10^{12}(v/c)^2 K \sim 10^{10} K$  for  $v = 0.1c$ ). However, the free-free cooling time is long in this regime –  $\sim 5 \times 10^8 n^{-1}$  years where  $n$  is the electron number density – and it exceeds the shock flow time at all interesting radii. For this reason, once the shell becomes energy-driven, it remains energy driven.